

Example 23.2:a) The system of *differential equations*

$$y_1' = \frac{y_1}{2 \cdot y_1 + 3 \cdot y_2}$$

$$y_2' = \frac{y_2}{2 \cdot y_1 + 3 \cdot y_2}$$

with the *initial conditions*

$$y_1(0) = 1, \quad y_2(0) = 2$$

has the exact solution

$$y_1(x) = \frac{x}{8} + 1, \quad y_2(x) = \frac{x}{4} + 2$$

The *numerical solution* using MATHCAD with the **rkfixed** *numerical function* is performed as follows:

$$\mathbf{y} := \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \mathbf{D}(x, \mathbf{y}) := \begin{pmatrix} \frac{y_1}{2 \cdot y_1 + 3 \cdot y_2} \\ \frac{y_2}{2 \cdot y_1 + 3 \cdot y_2} \end{pmatrix}$$

 $\mathbf{Y} := \mathbf{rkfixed}(\mathbf{y}, 0, 2, 10, \mathbf{D})$

$$\mathbf{Y} = \begin{pmatrix} 0 & 1 & 2 \\ 0.2 & 1.025 & 2.05 \\ 0.4 & 1.05 & 2.1 \\ 0.6 & 1.075 & 2.15 \\ 0.8 & 1.1 & 2.2 \\ 1 & 1.125 & 2.25 \\ 1.2 & 1.15 & 2.3 \\ 1.4 & 1.175 & 2.35 \\ 1.6 & 1.2 & 2.4 \\ 1.8 & 1.225 & 2.45 \\ 2 & 1.25 & 2.5 \end{pmatrix}$$

MATHCAD calculates in the *result matrix* \mathbf{Y} in the columns 2 and 3 the function values for the solution functions

$$y_1(x) \quad \text{and} \quad y_2(x)$$

for the x-values 0, 0.2, 0.4, ..., 2 of the first column.