Macierze i wektory - 1

Wprowadzamy wektory

$$v := \begin{pmatrix} 3 + 10 \\ 1 - 4 \\ 5 \cdot 10 \end{pmatrix} \qquad w := 2 \cdot v + \begin{pmatrix} 7 \\ 2 \\ -18 \end{pmatrix}$$

W wyniku otrzymujemy

$$v = \begin{pmatrix} 13 \\ -3 \\ 50 \end{pmatrix} \qquad w = \begin{pmatrix} 33 \\ -4 \\ 82 \end{pmatrix}$$

Mnożenie wektora przez skalar, dodawanie wektorów

$$3 \cdot v + w = \begin{pmatrix} 72 \\ -13 \\ 232 \end{pmatrix}$$

Wektor przeciwny do **w**

$$-w = \begin{pmatrix} -33 \\ 4 \\ -82 \end{pmatrix}$$

Wektor transponowany

$$v^T = \begin{pmatrix} 13 & -3 & 50 \end{pmatrix}$$

Iloczyn skalarny wektorów **v** oraz **w**

$$v \cdot w = 4.541 \times 10^3$$

Iloczyn wektorowy wektorów **v** oraz **w** (tylko dla dwóch wektorów trójelementowych)

$$v \times w = \begin{pmatrix} -46 \\ 584 \\ 47 \end{pmatrix}$$

Miara wektora **w** - jego długość (pasek narzędziowy [Calculator], przycisk $|x|$)

$$|w| = 88.482$$

Suma składowych

$$\sum v = 60$$

Dana jest macierz

$$M := \begin{pmatrix} 0 & 1 & 2 \\ 3 & 0 & 2 \\ 5 & 3 & 1 \end{pmatrix}$$

Macierz transponowana

$$M^T = \begin{pmatrix} 0 & 3 & 5 \\ 1 & 0 & 3 \\ 2 & 2 & 1 \end{pmatrix}$$

Wyznacznik macierzy **M**

$$|M| = 25$$

Potęgowanie macierzy

$$M^3 = \begin{pmatrix} 38 & 25 & 42 \\ 67 & 34 & 46 \\ 109 & 65 & 61 \end{pmatrix}$$

Macierz odwrotna

$$M^{-1} = \begin{pmatrix} -0.24 & 0.2 & 0.08 \\ 0.28 & -0.4 & 0.24 \\ 0.36 & 0.2 & -0.12 \end{pmatrix}$$

$$M^{-1} \cdot M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad M \cdot M^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Definiujemy elementy tablicy

$\alpha_0 := v \qquad \alpha_1 := M \qquad \alpha_2 := w$

Otrzymujemy

$$\alpha = \begin{bmatrix} \begin{pmatrix} 13 \\ -3 \\ 50 \end{pmatrix} \\ \begin{pmatrix} 0 & 1 & 2 \\ 3 & 0 & 2 \\ 5 & 3 & 1 \end{pmatrix} \\ \begin{pmatrix} 33 \\ -4 \\ 82 \end{pmatrix} \end{bmatrix}$$

[Result Format] - [Display Options] - [Expand nested arrays]

gdzie

$$\alpha_0 = \begin{pmatrix} 13 \\ -3 \\ 50 \end{pmatrix} \qquad \alpha_1 = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 0 & 2 \\ 5 & 3 & 1 \end{pmatrix} \qquad \alpha_2 = \begin{pmatrix} 33 \\ -4 \\ 82 \end{pmatrix}$$

$(\alpha_0)_{2,0} = 50 \qquad (\alpha_1)_{2,0} = 5$

$(\alpha_0)_2 = 50 \qquad \alpha_0$ traktowane jako wektor

WYKORZYSTANIE MACIERZY W OBLICZENIACH

$ORIGIN := 1$

$i := 1..10 \qquad j := 1..10$

$A_{i,j} := i^2 - j \qquad A_{5,5} := 999$

$$A = \begin{pmatrix} 0 & -1 & -2 & -3 & -4 & -5 & -6 & -7 & -8 & -9 \\ 3 & 2 & 1 & 0 & -1 & -2 & -3 & -4 & -5 & -6 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & -1 \\ 15 & 14 & 13 & 12 & 11 & 10 & 9 & 8 & 7 & 6 \\ 24 & 23 & 22 & 21 & 999 & 19 & 18 & 17 & 16 & 15 \\ 35 & 34 & 33 & 32 & 31 & 30 & 29 & 28 & 27 & 26 \\ 48 & 47 & 46 & 45 & 44 & 43 & 42 & 41 & 40 & 39 \\ 63 & 62 & 61 & 60 & 59 & 58 & 57 & 56 & 55 & 54 \\ 80 & 79 & 78 & 77 & 76 & 75 & 74 & 73 & 72 & 71 \\ 99 & 98 & 97 & 96 & 95 & 94 & 93 & 92 & 91 & 90 \end{pmatrix}$$

$B_{i,1} := \frac{(A_{i,1})^2}{A_{i,2}} \qquad B_{i,2} := B_{i,1} - 10 \qquad B_{i,3} := \frac{A_{i,5} - B_{i,2}}{4}$

$$B = \begin{pmatrix} 0 & -10 & 1.5 \\ 4.5 & -5.5 & 1.125 \\ 9.143 & -0.857 & 1.214 \\ 16.071 & 6.071 & 1.232 \\ 25.043 & 15.043 & 245.989 \\ 36.029 & 26.029 & 1.243 \\ 49.021 & 39.021 & 1.245 \\ 64.016 & 54.016 & 1.246 \\ 81.013 & 71.013 & 1.247 \\ 100.01 & 90.01 & 1.247 \end{pmatrix}$$

$$C(x,y) := \begin{pmatrix} x^2 & \frac{x}{y} & y \\ 1-x & x \cdot y^2 & 4 \\ 6 & y-x & x \end{pmatrix}$$

$$C(1,2) = \begin{pmatrix} 1 & 0.5 & 2 \\ 0 & 4 & 4 \\ 6 & 1 & 1 \end{pmatrix}$$

$$2 \cdot C(1,1) = \begin{pmatrix} 2 & 2 & 2 \\ 0 & 2 & 8 \\ 12 & 0 & 2 \end{pmatrix}$$

$$C(1,3)_{1,2} = 0.333$$

$$C(x,y)_{1,1} \rightarrow x^2$$

$$2 \cdot y \cdot C(x,y)_{2,2} \rightarrow 2 \cdot x \cdot y^3$$

$$z(x,y) := x^2 - x \cdot y \cdot C(x,y)_{2,1}$$

$$z\left(A_{2,3}, B_{3,3}\right) = 1$$

Insert - Data - Table (MathCad 11)

$D :=$

	1	2
1	14.45	2.8
2		
3		
4		
5		

Operator vectorize

$$v := \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} \qquad w := \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \qquad A := \begin{pmatrix} 1 & 2.4 & 0.7 \\ 0.5 & 3 & 8.4 \\ 0.35 & 2.54 & 4 \end{pmatrix}$$

$$i := 1 .. 4$$

$$\sin\left(i \cdot \frac{\pi}{12}\right) =$$

0.259
0.5
0.707
0.866

$$\sin(v) = \begin{pmatrix} 0.841 \\ -0.959 \\ 0.141 \end{pmatrix}$$

$$\sin(A) = \blacksquare$$

This must be either a scalar or a vector

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$$\overrightarrow{\sin(A)} = \begin{pmatrix} 0.841 & 0.675 & 0.644 \\ 0.479 & 0.141 & 0.855 \\ 0.343 & 0.566 & -0.757 \end{pmatrix}$$

$$v \cdot w = 18$$

$$\overrightarrow{(v \cdot w)} = \begin{pmatrix} 2 \\ 10 \\ 6 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2.445 & 11.378 & 23.66 \\ 4.94 & 31.536 & 59.15 \\ 3.02 & 18.62 & 37.581 \end{pmatrix}$$

$$\overrightarrow{A^2} = \begin{pmatrix} 1 & 5.76 & 0.49 \\ 0.25 & 9 & 70.56 \\ 0.122 & 6.452 & 16 \end{pmatrix}$$

Obliczenia symboliczne na wektorach i macierzach

$$A := \begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} \qquad v := \begin{pmatrix} v11 \\ v21 \\ v31 \end{pmatrix} \qquad w := \begin{pmatrix} w11 \\ w21 \\ w31 \end{pmatrix} \qquad B := \begin{pmatrix} a11 & a12 \\ a21 & a22 \end{pmatrix}$$

$$A \cdot v \rightarrow \begin{pmatrix} a11 \cdot v11 + a12 \cdot v21 + a13 \cdot v31 \\ a21 \cdot v11 + a22 \cdot v21 + a23 \cdot v31 \\ a31 \cdot v11 + a32 \cdot v21 + a33 \cdot v31 \end{pmatrix}$$

$$B^{-1} \text{ simplify } \rightarrow \begin{pmatrix} \frac{a22}{a11 \cdot a22 - a12 \cdot a21} & -\frac{a12}{a11 \cdot a22 - a12 \cdot a21} \\ -\frac{a21}{a11 \cdot a22 - a12 \cdot a21} & \frac{a11}{a11 \cdot a22 - a12 \cdot a21} \end{pmatrix}$$

$$B^2 \rightarrow \begin{pmatrix} a11^2 + a12 \cdot a21 & a11 \cdot a12 + a12 \cdot a22 \\ a11 \cdot a21 + a21 \cdot a22 & a22^2 + a12 \cdot a21 \end{pmatrix} \qquad B^T \rightarrow \begin{pmatrix} a11 & a21 \\ a12 & a22 \end{pmatrix}$$

$$v \cdot w \rightarrow v11 \cdot \overline{w11} + v21 \cdot \overline{w21} + v31 \cdot \overline{w31}$$

$$v \times w \rightarrow \begin{pmatrix} v21 \cdot w31 - v31 \cdot w21 \\ v31 \cdot w11 - v11 \cdot w31 \\ v11 \cdot w21 - v21 \cdot w11 \end{pmatrix}$$

$$C := \begin{pmatrix} c & 4 \\ a & c^2 \end{pmatrix} \qquad C^2 \rightarrow \begin{pmatrix} c^2 + 4 \cdot a & 4 \cdot c^2 + 4 \cdot c \\ a \cdot c^2 + a \cdot c & c^4 + 4 \cdot a \end{pmatrix}$$